QUANTUM TIME

By

Jonathan A. Oppenheim

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Department of Physics and Astronomy
The University of British Columbia
2075 Wesbrook Place
Vancouver, Canada
V6T 1W5

Date:

Abstract

In quantum mechanics, time plays a role unlike any other observable. We find that measuring whether an event happened, and measuring when an event happened are fundamentally different – the two measurements do not correspond to compatible observables and interfere with each other. We also propose a basic limitation on measurements of the arrival time of a free particle given by $1/\bar{E}_k$ where $\bar{E}_k$ is the particle's kinetic energy. The temporal order of events is also an ambiguous concept in quantum mechanics. It is not always possible to determine whether one event lies in the future or past of another event. One cannot measure whether one particle arrives to a particular location before or after another particle if they arrive within a time of $1/\bar{E}$ of each other, where $\bar{E}$ is the total kinetic energy of the two particles. These new inaccuracy limitations are dynamical in nature, and fundamentally different from the Heisenberg uncertainty relations. They refer to individual measurements of a single quantity. It is hoped that by understanding the role of time in quantum mechanics, we may gain new insight into the role of time in a quantum theory of gravity.

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Dedication

In loving memory of my father, Peter Oppenheim (1942-1998) - my first physics teacher, who encouraged my curiosity, patiently answered my questions, and patiently asked his own. He would have loved to flip through this thing, and I had always imagined giving him a copy.

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Chapter 1

Introduction

1.1 Dual Measurements

One of the first lessons of quantum mechanics was that a property of a system does not correspond to an element of reality until it is measured. It makes no sense to talk about the position of a particle or the momentum of the particle, in and of itself. It is only when we measure a physical quantity that we can actually say that a system possesses it. The particle does not have a position until its position is actually measured.

Ordinarily in quantum mechanics, one is interested in measuring properties of a system at a particular time t. One might want to know a particle's position, momentum, or spin, and the measurement of this quantity occurs at a certain time. For experiments at a fixed time, quantum mechanics provides us with a useful formalism to describe reality. Observables are represented by self-adjoint operators, and in the Heisenberg representation they evolve in time. The possible results of any measurement at any instant of time t can be found by applying these operators to the wave function of the system at the time t.

This immediately raises the question of what the parameter time t represents in the Heisenberg equations of motion. Since t is a number and not a self-adjoint operator, it does not appear to be an observable in the usual sense. For any measurement of an observable $\mathbf{A}(t)$ of a system, one can imagine a *dual* measurement, where one attempts to measure the time t_A at which the system attains a particular value of $\mathbf{A}$. The dual measurement determines the time a certain event occurs, where the event in question is the system attaining a particular value (or values) of an observable. For example, instead of measuring the position of a particle at a certain time, one can consider the dual measurement where the roles of x and t are interchanged. Instead of measuring where the particle is at time t, one measures the time that a particle is found at a particular location x_A. In this dual measurement, the position x_A is the parameter while the time becomes the observable one is trying to measure.

Classically, the time of an event can be made into an observable just like any other and this time can be measured in a variety of ways, all of which give the same result. One can simply invert the equations of motion of the system to find the time that an event occurs [1], and then measure the values of the canonical variables (generalized coordinates and conjugate momenta). Since classically there is no uncertainty relation preventing the measurement of all the coordinates and conjugate momenta simultaneously, there is no limitation for finding the event's time. One could also continually monitor the system to determine the precise time when the event occurred. Since one can make the interaction between the system and the measuring apparatus as small as one likes, this measurement need not disturb the evolution of the system. Finally, one can also couple a clock to the system in such a way that the clock stops when the event occurs. All these methods yield the same results, and work to any desired accuracy.

Dual measurements, are quite common in modern laboratory experiments. In particle physics one often wants to know the time that certain collisions or decays occurred. However, surprisingly, dual measurements are not easily dealt with using the conventional tools of quantum mechanics.

Pauli [8] was the first to demonstrate that there was no operator associated with time. A time operator must be conjugate to the Hamiltonian, and he proved that this is impossible if the Hamiltonian for the system is bounded from above or below. The reason for this is that an operator which is conjugate to the Hamiltonian acts as a shift operator for energy, and one could use it to shift the energy below any lower bound (or above any upper bound).

Since then, there have been numerous attempts to circumvent his proof by considering

[1] In some systems (especially in the context of general relativity), it is only possible to find the time locally. A global time variable may not exist.

either modified time operators, or by considering operators which correspond to the "time-of-arrival", i.e. the time that an event first occurs [9][12]. At first glance, the latter operator need not be conjugate to the Hamiltonian, since all that is required is that the time-of-arrival operator not evolve in time [2]. However, in Chapter 4 we show that, in general, time-of-arrival operators also do not exist.

Aharonov and Bohm were the first to write down a time-of-arrival operator [10], and since then most of the work in the field of time-of-arrival has involved interpreting, or modifying this operator[11][12], or operators associated with it (such as the "current operator"[13]). Allcock [14][15] was the first to examine a physical model for measuring the time-of-arrival, and this led him to suggest that time-of-arrival may not be measurable. However, his model did not contain a clock, and he argued that the source of the difficulty in measuring time-of-arrival was in absorbing a particle in an arbitrarily short length of time. In fact, as we will discuss, the source of the difficulty lies elsewhere, and one needs to use models with physical clocks. Peres [16], has used physical clocks to describe measurements of various quantities, although not in the context of time-of-arrival, and physical clocks have also been discussed in the context of barrier tunneling time [17].

Although much of the work in this field (including our own) is done in the context of the Copenhagen interpretation of quantum mechanics, the problem of time-of-arrival has also been studied in the context of the decoherent histories formalism [18]. In this approach, amplitudes are not assigned to events at a certain time, but rather to entire histories in a decohering set of histories. For the case of time-of-arrival, one finds that histories which correspond to different arrival times do not decohere unless the particle is coupled to an environment or a model detector [19]. In many respects, this supports the approach that one must consider physical measurement processes in order to measure

[2] by definition, the arrival time is the same at all times - if I will (or did) arrive at 5 p.m., then this statement remains true at 3 p.m., 4 p.m. and 6 p.m.

the time-of-arrival.

The interest in a quantum mechanical time operator stems in part from the troubling notion that elements of reality should be observables. It is hard to understand what the parameter t in the Schrödinger equation means, if it does not correspond to something physically measurable. Recently however, attempts to find a quantum theory of gravity have also inspired numerous authors to examine the problem of time in quantum mechanics.

In general relativity (and even in many laboratory experiments), we are often interested in performing experiments which are not fixed in time. For example, if we wish to measure space-time distances, then we will probably want to know how long it takes for a photon to travel between two points. This is a continuous measurement which does not occur at any particular time. We may also want to know whether one event is in the past or future of another event. Both these measurements are, in some sense, a measurement of time itself, and it is these types of measurements which are necessary in order to determine the components of the metric tensor.

Another physical property, which appears in the context of quantum cosmology, is the maximum size the universe will attain. This is not a property of the universe at a fixed time, but rather, a property of the universe over all time. In classical physics, one could make measurements on a system at a fixed time in order to predict the evolution of the system for all time. However, as we will see, in quantum mechanics this is not always the case.

It is widely believed that one of the difficulties of constructing a quantum theory of gravity, is that time plays an incompatible role in quantum mechanics and general relativity [20]. In quantum mechanics, time is an external parameter while in general relativity, time is much more a part of the theory. Both time and space bend and twist in the presence of massive objects, and both space and time are represented by coordinates.

It is space-time which is the element of reality in general relativity.

These coordinates are, of course, subject to coordinate transformations, and in particular, the theory is invariant under reparametrization of the time coordinate. One consequence of this, is that if one tries to canonically quantize Einstein's theory of gravity in a closed system, one finds that the wave-function must satisfy the Wheeler-DeWitt equation

$$\mathcal{H}\Psi(g_{ab}, \pi_{ab}) = 0 \tag{1.1}$$

where the wave function depends on the 3-metric and conjugate momenta and $\mathcal{H}$ is known as the Hamiltonian constraint and is the generator of time reparametrizations and time-translations when the equations of motion are satisfied. Because the Hamiltonian constraint must always be satisfied, most standard interpretations require that the only possible observables are those which commute with the constraint [22]. However, observables which commute with the constraints don't evolve in time, making the system rather hard to describe. It is not clear how to best frame physically meaningful questions, if all the observables are static with respect to parameter time.

One of the central sources of the the problem of time in quantum gravity (and quantum cosmology in particular) is that it attempts to describe the entire universe quantum mechanically. There is no external observer, and therefore, no external parameter time. Many authors have therefore attempted to develop alternative frameworks of quantum mechanics which do not rely on an external time parameter [18][20][22][21]. By re-examining the way in which we think about time, we may be able to construct a consistent theory of quantum gravity. In this thesis, we make no pretense of trying to solve the problem of time in quantum gravity. Rather, we take the approach that in order to understand the role of time in quantum gravity, one must first understand the role of time in quantum mechanics. As it turns out, this is far from easy, and there still exist many ambiguities in the role of time in quantum mechanics. Our hope is that a better understanding of time in the arena of quantum mechanics will benefit and inform research in the field of quantum gravity. At the end of this thesis, we will discuss some of the connections between the problem of time in quantum gravity and our research.

1.2 Differences Between Measurements of Space and Measurements of Time

Ever since Einstein's theory of special relativity, we have been encouraged to think of time and space on an equal footing. However, even classically, time and space are quite different as our common experience tells us. Objects move constantly forward in time an a manner very different to the way they move through space. Although we will discuss in more detail the differences between quantum measurements of ordinary observables and measurements of time in Chapter 2, it may be instructive to roughly outline the differences between measurements of a particle's position at a fixed time, and the time a particle is found at a particular location.

In standard quantum mechanics, the probability that a particle is found at a given location X at time t is given by

$$P_t(X) = |\psi(X,t)|^2 \quad . \tag{1.2}$$

If we know $\psi(x,0)$ for all x then the system is completely described and we can easily compute this probability distribution at an instant of time. If we know the Hamiltonian of the system, then using the Schrödinger equation we can also compute $\psi(x,t)$ at any time t. This probability distribution corresponds to results of a measurement of position at a particular time. Quantum mechanics gives a well defined answer to the question, "where is the particle at time t?"

However, it is also perfectly natural to ask "at what time is the particle at a certain location." Here, quantum mechanics does not seem to provide an unambiguous answer.

At first sight it seems that the probability distribution $P_x(T)$ to find the particle at a certain time at the location x is simply $|\psi(x,T)|^2$, However, $|\psi(x,T)|^2$, does not represent a probability *in time*, since it is not normalized with respect to T.

One might be tempted therefore, to consider the quantity

$$P_x(T) = \frac{|\psi(x,T)|^2}{\int |\psi(x,t')|^2 dt'} \tag{1.3}$$

This normalization depends on the particular state being measured, and can only be done if one knows the state $\psi(x,t)$ at all times t (infinitely far in the past and future). There are also states for which the particle is never found at the position x, in which case the expression above is undefined. Not-withstanding this, one might argue that this quantity gives one a relative probability that the particle is found at the location x at time T (if the measurement is made at that time T), as opposed to another time T' (if the measurement is made at time T').

However, the expression above certainly does not yield the probability *in time* to detect the particle. One reason for this failure is that a particle may be detected at a location X at many different times t (e.g. I can be found in my office at many different times in the day). On the other hand, if at time t a particle is detected at location X, then we can say with certainty that at the same time t, the particle was not at any other location X' (e.g. at nine a.m. I am in bed, and therefore, I cannot also be in my office). Equation (1.3) does not give a proper probability distribution as the various outcomes are not disjoint. $P_x(T)$ is not a probability distribution in time in the sense usually reserved for probability distributions in quantum mechanics. $P_x(T)$ is very different from $P_t(X)$ and has different properties (as we will see in the next chapter).

This leads us to consider the time of first arrival of a particle, since a particle can only arrive once to a particular location. In order to measure the arrival time one cannot use expression (1.3) since one needs to detect the particle at time t_A, and also know that the particle was not there at any previous time. In other words, one must continuously monitor the location x_A in order to find out when the particle arrives. However, this continuous measurement procedure has it's own difficulty, and also emphasizes the problem with the previous probability distribution. Namely, that the probability to find a particle at $t = T$ is generally *not* independent of the probability to find the particle at some other time $t = T'$. i.e.. if $\mathbf{\Pi}_{x_A}$ is the projector onto the position x_A, then in the Heisenberg representation [3]

$$[\mathbf{\Pi}_{x_A}(t), \mathbf{\Pi}_{x_A}(t')] \neq 0. \tag{1.4}$$

Measurements made at different times disturb each other. We will see in Section 2.2 that this is one of the properties of ordinary measurements which measurements in time violate. Measurements made at different times do not commute. Therefore the probability distribution obtained from this measurement procedure, although well defined, does not give a probability distribution *in time.*

Von Neumann measurements [4] happen *at a certain time.* One measures the particle's position at time t. Even a continuous measurement at a particular location is a series of measurements at a certain time. Each instant that the Geiger counter doesn't click, it is measuring the fact that a particle has not entered it. Furthermore, operators which are used to measure the time-of-arrival to the location x_A, are not measured at x_A, but rather at an instant in time. In quantum mechanics, measurements made at different times can disturb each other, which can make measurements of the time of an event problematic. The probability of detecting a particle at a certain location at time t is not independent

[3] The exact expression for the commutator of the position projectors at different times is not particularly illuminating. However, it is fairly obvious $\mathbf{\Pi}_{x_A}$ doesn't commute with itself at different times, because the position operator itself doesn't commute with itself at different times. I.e., since $\mathbf{x}(t) = \mathbf{x}_0 + \mathbf{p}t/m$, we have $[\mathbf{x}(t), \mathbf{x}(0)] = it/m$.

[4] In [23] Von Neumann outlined how one goes about measuring observables which correspond to self-adjoint operators. The results one obtains for the measurements are universal, and correspond to actual properties of the system.

of detecting the particle at some other time t'.

1.3 Inaccuracies and Uncertainties

The measurement of an observable corresponding to a self-adjoint operator can be as accurate as one wishes. This is true despite any uncertainty relations which govern various sets of observables. The position, or momentum of a particle (but not both) can be measured to any desired precision. Consider two observables **A** and **B** which do not evolve in time, and whose commutator is i (in units where $\hbar = 1$). Imagine that we have an ensemble of identical systems prepared in some initial state. On half the ensemble, we can measure **A**, and on the other half, we can measure **B**. Each individual measurement can be as accurate as we wish. An extraordinary experimentalist can reduce the inaccuracies in the measurement to almost zero, and can get a particular value for each measurement. The experimentalist may have a dial on her device which will point to the value of A after the measurement. She will have to make sure that initially the pointer on her dial points almost exactly to zero, and then after each run of her experiment, she measures the position of the dial very accurately to determine the value of A.

If we then plot all of the measurements of **A** and all of the measurements of **B**, we will find a distribution of measurements which have a natural width of ΔA and ΔB respectively. One then finds that no matter what initial state we choose, $\Delta A \Delta B > 1$. There is an *uncertainty* relation between the distributions of A and B, but there are no theoretical limitations on the accuracy of each individual measurement of **A** or **B**.

The experimentalist does not have to make her measurements totally precise. She could, for example, start off the experiment with her dial in a state where the initial position of the needle is *uncertain*. An uncertainty in the initial pointer position will result in her measurement being *inaccurate*. When she measures the final position of her pointer, she will not be able to infer the precise value of the measurement of **A** or **B** because she will not know exactly what the initial value the pointer was set to.

For measurements of conventional observables, there are no limitations on the inaccuracy of measurements. However, we will find that for certain observables relating to the time-of-events, one must make the measurement inaccurate. If one attempts to make the measurement too accurate, one finds that the measurement fails.

The inaccuracy limitations we find are not equivalent to the so-called "Heisenberg energy-time uncertainty principle". The limitations refer to individual measurements of a single quantity. Quantum mechanics places no limitation on how accurately we can make a single measurement of position or momentum (although an accurate measurement of position will disturb the momentum and visa-versa). For measurements of time-of-arrival however, we cannot make a single measurement arbitrarily accurate. If we do so, we may find that the particle never arrives.

1.4 What Lies Ahead

In this thesis, we will find that dual measurements are fundamentally different from measurements of ordinary quantum variables. We will examine a number of different types of dual measurements as well as various methods for making them. In Chapter 2 we will look at measurements where one continually monitors the state to determine whether the event has occurred. This involves a series of measurements at closely spaced time intervals. We find the surprising result that the question of whether an event *has* occurred and the question of *when* it occurred are not compatible observables. Describing attributes of a system *in time* is fundamentally different from describing attributes at a *given time*. The more difficult question of "when did the event occur?" cannot be measured unambiguously in quantum mechanics. We also critically examine the use of the probability current to measure the time at which a particle arrives to a certain location. The discussion suggests that the difference between time and other observables is not merely formal.

The central result of the thesis is contained in Chapter 3 where we discuss the problem of the time-of-arrival of a particle to a particular location. It is argued that the time-of-arrival cannot be precisely defined and measured in quantum mechanics. By constructing explicit toy models of a measurement involving physical clocks, we show that the time-of-arrival for a free particle cannot be measured more accurately then $\delta t_A \sim 1/\bar{E}_k$, where $\bar{E}_k$ is the initial kinetic energy of the particle. With a better accuracy, particles reflect off the measuring device and the resulting probability distribution becomes distorted. This is a new relation which is not equivalent to the so-called "Heisenberg energy-time uncertainty" [5] - it places a restriction on each individual measurement of time-of-arrival.

The basic reason for the inaccuracy limitation is that while one can construct an arbitrarily accurate clock, using this clock presents difficulties. The more accurate the clock, the greater the spread in the clock's energy. Accurate clocks are extremely energetic, and this makes it harder for the system to stop the clock. In order to use the clock to measure the time of an event, one needs the system to turn off the clock when the event occurs. For accurate clocks, the system will not always have enough energy to turn it off, and no measurement will occur.

Recently, many authors [12] have attempted to construct operators which can be used to measure the time-of-arrival of a particle. In Chapter 4 we present a formal proof that a time-of-arrival operator cannot exist. Still, many believe that one can modify a time-of-arrival operator in such a way as to make the concept useful. We discuss the

[5] For convenience, we will sometimes use the term "Heisenberg's energy-time relation". It should be remembered however, that since time is not represented by a self-adjoint operator, the uncertainty relation is actually between energy, and observables of the system which evolve in time (for example, an atom's life-time becomes uncertain if the atom is close to an eigenstate of energy)

relationship between these modified operators, and the direct measurements discussed in Chapters 2 and 3, and argue that a measurement of the time-of-arrival operator does not correspond to these continuous measurements. Unlike the classical case, in quantum mechanics the result of a measurement of the time-of-arrival operator may have nothing to do with the time-of-arrival to $x = x_A$.

There has been renewed interest in time-of-arrival operators following the suggestion by Grot, Rovelli, and Tate, that one can modify the low momentum behavior of the operator slightly in such away as to make it self-adjoint [9]. We show that such a modification results in the difficulty that the eigenstates are drastically altered. In an eigenstate of the modified time-of-arrival operator, the particle, at the predicted time-of-arrival, is found far away from the point of arrival with probability 1/2.

The bound of $1/\bar{E}_k$ on the accuracy of time-of-arrival measurements is based on calculations done using numerous measurement models corresponding to specific Hamiltonians, as well as more general considerations. However, because the limitation is based on dynamical considerations and not kinematic ones, a formal proof of the limitation may not exist. For example, a proof of the Heisenberg uncertainty relation relies only on the properties of specific operators, while our inaccuracy relation is a statement not about operators, but about measurements (and therefore, involves the dynamical considerations of the actual measurement). Perhaps by making certain restrictive assumptions about the Hamiltonian one might be able to construct a formal proof. Such a proof would have to take into account the measurement model which will be discussed in Section 3.3.3 in which we show that if one has prior information about the wavefunction, and if the wavefunction is almost an eigenstate of energy (i.e. its time of arrival is completely uncertain), then one can measure the time of arrival to an accuracy better than $1/\bar{E}_k$. One therefore expects that a formal proof will not only have to involve making assumptions about the interaction Hamiltonian, but also the initial state of the wave function. The existence of a formal proof for our inaccuracy limitation remains an interesting open question.

While we know of no formal proof for the inaccuracy limitation for time-of-arrival, one can make more general statements about measurements of "traversal time". In Chapter 5 we consider the problem of a free particle which traverses a distance L and argue that a violation of the above limitation for the traversal-time implies a violation of the Heisenberg uncertainty relation for x and p. This result does not depend on the details of the model being used in the measuring process. Measurements of traversal-time are dual to measurements of traversal distance, and it can be shown that one can measure the distance a particle travels to any desired precision. This chapter also contains a further discussion on the difference between what we call "inaccuracy" limitations, which constrain the precision with which individual measurements are performed, and "uncertainties" which are kinematic quantities which relate to the spread in measurements on ensembles.

Chapter 6 contains what may be our most interesting result. In it, we examine whether one can determine the temporal ordering of events. We find that one cannot measure whether one event occurred in the future or past of another event to arbitrary accuracy. The minimum inaccuracy for measuring whether a particle arrives to a given location before or after another particle is given by $1/\bar{E}$ where $\bar{E}$ is the total kinetic energy of the two particles. We discuss the relationship between this type of measurement, and coincident counters, as well as Heisenberg's microscope. We show that in general one cannot prepare a two particle state where the two particles always arrive within a time of $1/\bar{E}$ of each other. This has interesting consequences for determining the metric properties of a space-time.

In this thesis we will work in units where $\hbar = c = 1$