Resource Theories

Quantumness & Thermodynamics

J. Oppenheim

J. O. Horodecki$^{\otimes 3}$ quant-ph/0112074

Horodecki$^{\otimes 3}$, J O, Sen$^{\otimes 2}$, Synak quant-ph/0410090

Horodecki, J.O. arXiv/1209.2162

My Motivations

Understanding structure of Resource Theories

I like thermodynamics and its connection with information

- quantum thermodynamics
  many second laws

Outline

Resource Theories

Discord as a resource theory?

Complementarity of local, nonlocal Info

Thermodynamics as a R.T.

Correlations in thermo

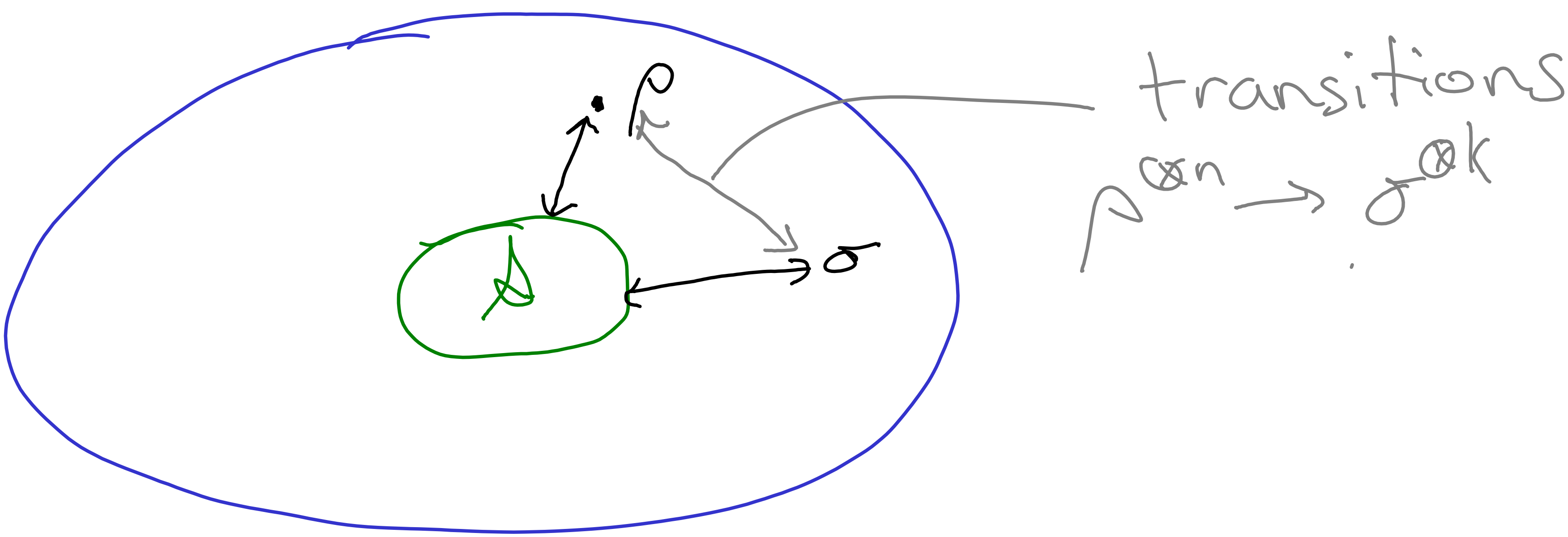
Resource Theories

Class of operations $\mathcal{C}$

Free set of states $\mathcal{S}$ &

Resources $(\neg \mathcal{S})$

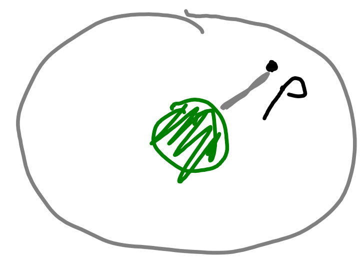
Distance to that set $\mathcal{S}$



Distance to the Free States

eg $S(\rho\|\sigma) = -\mathrm{tr}\,\rho\log\sigma + \mathrm{tr}\,\rho\log\rho$

$M(\rho) := \inf_{\sigma\in\mathcal{S}} S(\rho\|\sigma)$



Generic measure under the class $\mathcal{C}$
(monotone)

Unique measure if $\mathcal{C}$ reversible, & convex

- Horodecki, J.O.
quant-ph/0207177 /1209.2162

We can find a reversible $\mathcal{C}$
- Brandao, Plenio arxiv/0710.5827
- Brandao, Gour arxiv/1502.03149

Eg Entanglement theory

Class of Operations: LOCC

Free States: Seperable

Resource: entanglement (singlet standard)

Transitions: $\rho_{AB}^{\otimes n} \longrightarrow \sigma_{AB}^{\otimes k}$

Distance: to σ_{sep} as entanglement measure

E_R (plenio & vedral)

"unique measure"

Eg Purity theory

Class of Operations: Noisy Ops

- adding noise $\mathbb{I}/d$
- unitaries
- tracing out

H^2O : quant-ph/0212019

Free States: $\mathbb{I}/d$

Resource: pure states

Distance to $\mathbb{1}/d$: $D(\rho \| \frac{\mathbb{I}}{d}) = N - S(\rho)$

Transitions: $\rho^{\otimes n} \longleftrightarrow \sigma^{\otimes k}$

Purity theory

Eg. compression

$$\underbrace{\rho^{\otimes n}}_{n} \longrightarrow \approx \underbrace{\tau^{(nS(\rho))}}_{nS(\rho)} \otimes \underbrace{|0\rangle\langle 0|^{\otimes n(1-S(\rho))}}_{n(1-S(\rho))}$$

instead of keeping τ and throwing away $|0\rangle$

we keep the purify and throw away signal

$$\rho^{\otimes n} \longleftrightarrow \sigma^{\otimes k}$$

$$\frac{n}{k} = \frac{N - S(\sigma)}{N - S(\rho)}$$

Thermo for children

Eg Thermodynamics

Class of Operations : Thermal Ops

- adding free states with Hamiltonian H_R

① - unitaries which conserve energy

- tracing out

⓪ Free States : Thermal states $\tau = e^{-\beta H_R}/Z$

Resource : work, athermal states

② Distance to $e^{-\beta H_R}/Z$ is $F = E - TS$, F_α

Transitions : $\rho \longleftrightarrow \sigma$

Janzig et al quant-ph/0002048, HO quant-ph/1111.3834

Eg Discord / Deficit

Class of Operations: NLOCC*

- adding local noise I/d
- local unitaries
- tracing out
- local dephasing and sending

0 way
1 → way
2 ↔ way

Free States: classically correlated?

Resource: local purity I_ℓ

Distance to classical states?

Δ is not closed under ⊗ except 0-way

Transitions: $\rho_{AB}^{\otimes n} \rightarrow \sigma_{AB}^{\otimes k}$: #CTO

Resource

Entanglement: distill singlets

$E_d(\rho_{AB})$, E_f

seperable states free

Discord/Deficit: distill local purity

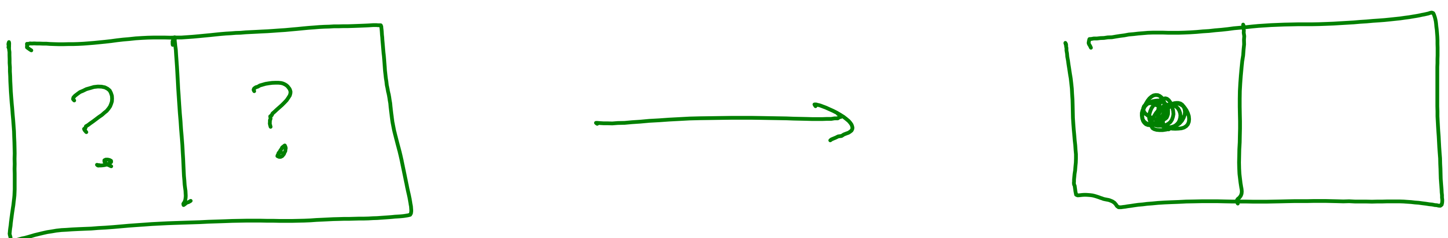
$I_e(\rho_{AB})$, I_f (kinda)

noisy states $\mathbb{1}/d$ are free

Purity Theory is like Thermodynamics

Reducing the entropy of ρ (distilling pure states) requires work

Landauer erasure



$W = kT \log 2$

I_ℓ : Local work extraction

Horodecki$^{\otimes 3}$, JO, Sen$^{\otimes 2}$, Synak   quant ph/0410090
(also discovered by Modi, Vedral 2011)   quant ph/0207168

$\mathcal{S}$		$M(\rho) := \inf_{\sigma \in \mathcal{S}} S(\rho \Vert \sigma)$
$\mathcal{S}^{all}$	$\mathbb{1}/d$	N-S
$\mathcal{S}^{0}$	"classical" states	Δ^{0}
$\mathcal{S}^{1}$	c-q states	$\vec{\Delta}$ → monotone when Δ_{cl} regularised
$\mathcal{S}^{2}$	IPP states	Δ?
$\mathcal{S}^{sep}$	sep	$E_R(\rho)$ Plenio, Vedral
$\mathcal{S}^{g}$	g	frameness (Gour, Spekkens)
$\mathcal{S}^{thermal}$	gibbs	$F = E - TS$

others: coherence, magic state distillation

Resource Theories

Theory	Entanglement	Purity	Asymmetry
Class of Operations $\mathcal{C}$	LOCC	Noisy $\frac{\mathbb{1}}{d}$, U	$T(g)\circ\mathcal{E}\circ T^{\dagger}(g) = \mathcal{E}$
Free states $\mathcal{S}$	seperable σ	$\mathbb{1}/d$	$\sigma = T(g)\sigma T^{\dagger}(g)$
Monotones	$\inf_{\sigma} M(\rho\|\sigma)$	$M(\rho\|\frac{\mathbb{1}}{d}) = N - S(\rho)$	$\inf_{\sigma} M(\rho\|\sigma)$

"Discord"
Deficit

Thermodynamics

Purity Extraction under Different Classes

C	I (purity)	
NO	$I = N - S(\rho_{AB})$	Δ_q
NLOCC	$I_c = N - S(\rho'_{AB})$	I_M
NLO	$I_{LO} = N - S(\rho_A) - S(\rho_B)$	Δ_c

$$\Delta_q = I - I_c$$

$$\Delta_c = I_c - I_{LO}$$

$$I_M = I - I_{LO}$$

Comparisons!

Complementarity of Local and Nonlocal Info

$O, H^{\otimes 3}$ quant-ph/0207025 (rediscovered by Zwolak, Zurek) 2013

$I = I_c + \Delta_q$

$I(\Psi^+) = 1 + 1 = 2$

Total information = classical + quantum

or!

Either extract local purity or teleport (nonlocal info)

$I(\Psi^+) = 2 \neq 1 + 1 \qquad 2 = 1 \text{ or } 1$

Thermodynamics as a resource theory

Class of Operations	Thermal	Energy conserving U Heat bath τ at temp T Work as $\|0\rangle \to \|W\rangle$
Free Set	Heat bath τ at temp T	
Monotones	$F = D(\rho \| \tau)$	$\to F_{max}$ $\to F_{min}$

Janzig et al quant-ph/0002048

(HO, 2011) quant-ph/1111.3834

3 laws of thermodynamics

0) If R_1 is in equilibrium with R_2 and R_3 then R_2 is in equilibrium with R_3

1) $dE = dQ - dW$ (energy conservation)

2) Heat can never pass from a colder body to a warmer body without some other change occuring. –Clausius

3) One can never attain $T=0$ in a finite number of steps

2 law of quantum thermodynamics

0) The only free state σ which doesn't enable arbitrary transitions is the thermal state γ

2)* The state ρ_S must get closer to γ_S in terms of free energy type distances

$F_\alpha(\rho_S \| \gamma_S) \quad \forall \quad 0 < \alpha$

* depends on how cyclic

3) Number of steps to attain any transition.

Masanes, JO 1412.3828

Thermodynamics

Many Second Laws!	(HO, 2011) arXiv/1111.3834 (BHNOW, 2013) arXiv/1305.5278
Second laws for coherences	Cwiklinski et al 1405.5029 Lostaglio et al 1405.2188 Lostaglio et al 1410.4572
Correlating systems allows you to do work!	Mueller, Pastena 1409.3258
Correlating systems costs work	Bruschi et al 1409.4647 Perarnau-Llobet 1407.7765

Relationship with other measures

$\overleftarrow{\Delta}_c \leq \Delta_{H.V.}$

but regularizations equal (Devetak 2005)

$D_{th} = \overleftarrow{\Delta}_q$ (Zurek 2003)

D_{th}: δ optimised over VN measurement

Conclusions & Open Questions

Discord as:

- a resource theory (extracting purity)
- entropy generation
- local thermodynamics

Relative entropy distance to free states

Complementarity of local, nonlocal info

Open Questions quant-ph/0410090

e.g. $\overleftrightarrow{\Delta}_q$ as relative entropy distance?

Thermodynamics

- correlations and work
- additional second laws for coherences?
- extracting work from correlations
- understanding catalytic operations
- Thank you for your attention